# CONTINUATIO CIRCULI QUADRATURÆ NOVISSIMÆ ET BREVISSIMÆ.



## PROBLEMA IV.

§. 13. *Determinare partes constituentes summam, cujus numerator est aggregatum ex utroque factore denominatoris.*

Sit denominator summæ $= mo$ & numerator $= o + m$, h. e. sit ille aggregatum ex utroque factore $o$ & $m$ denominatoris $mo$: erit summa ipsa $= \frac{o+m}{mo} = \frac{o}{mo} + \frac{m}{mo} = \frac{1}{m} + \frac{1}{o}$

Theorema: *Si numerator summæ est aggregatum ex utroque factore denominatoris; necesse est, ut unus factor sit denominator unius partis, alter factor denominator alterius, & unitas numerator utriusque partis.*

§. 14. *Corollarium.* Si itaque summa fuerit $= \frac{d+a}{ad}$; erunt partes constituentes excessum & defectum $\frac{1}{a}$ & $\frac{1}{d}$, & quidem (per §4.) prior excessus & posterior defectus.

§. 15. Scholion. *Hanc veritatem esse indubitatam, colligi potest ex investigatione lunulæ respondentis diam. quadrato* $= 1$, *quam suppono nondum esse inventam. Sit itaque ratio hujus quadrati ad lunulam excessiva, ut* $4 : 1\frac{1}{2} = 8 : 3$ *& defectiva ut* $4 : \frac{2}{3} = 12 : 2$; *erunt lunulæ* $\frac{3}{8}$ *&* $\frac{2}{12} = \frac{36}{96}$ *&* $\frac{16}{96}$: *consequenter* $\frac{35}{96} - \frac{16}{96} = \frac{20}{96}$ *summa excessus & defectus* (§ 2.), *cujus numerator est* $= d+a = 12+8$. *Ergo* (*per* § 4.) *excessus est* $\frac{1}{8}$ *& defectus* $\frac{1}{12}$. *Sit deinde ratio excessiva, ut* $4 : 1\frac{1}{6} = 24 : 7$ *& defectiva, ut* $4 : \frac{6}{7} = 28 : 6$: *erunt lunulæ* $\frac{7}{24}$ *&* $\frac{6}{28} = \frac{196}{672}$ *&* $\frac{144}{672}$: *consequenter* $\frac{196}{672} - \frac{144}{672} = \frac{52}{672}$ *summa excessus & defectus, cujus numerator est* $= d+a = 28+24$. *Ergo excessus est* $\frac{1}{24}$ *& defectus* $\frac{1}{28}$. *Jam cum hi excessus subducti ex lunulis excessivis, & defectus additi ad defectivas, manifestent utrobique lunulam* $= \frac{1}{4}$: *consequenter rationem ejus ad diam. quadratum eandem, quæ reperitur per Theorema* Hippocratis; *dubitari nequit, quin excessus & defectus per Problema III. legitime determinentur. Pergamus jam à cognitis ad incognita: Sit ratio quadrati diametri* $= 1$ *ad segmentum ei respondens excessiva, ut* $64 : 9\frac{1}{8} = 512 : 73$

&

*& defectiva, ut* $64 : 8\frac{3}{4} = 256 : 35$ *: erunt segmenta* $\frac{73}{512}$ *&* $\frac{35}{256} = \frac{18688}{131072}$ *&* $\frac{17920}{131072}$, *consequenter* $\frac{18688}{131072} - \frac{17920}{131072} = \frac{768}{131072}$ *summa excessus & defectus, cujus numerator est* $= d + a = 256 + 512$. *Ergo excessus est* $\frac{1}{512}$ *& defectus* $\frac{1}{256}$*: consequenter segmentum verum* $\frac{72}{512} = \frac{9}{64}$ *vel* $\frac{36}{256} = \frac{9}{64}$ *& ad quadratum diam. ut* $\frac{9}{64} : 1 = 9 : 64$. *Sit deinde ratio excessiva, ut* $64 : 9\frac{1}{4} = 256 : 37$ *& defectiva, ut* $64 : 8\frac{2}{3} = 192 : 26$ *: erunt segmenta* $\frac{37}{256}$ *&* $\frac{26}{192} = \frac{7104}{49152}$ *&* $\frac{6656}{49152}$ *: consequenter* $\frac{7104}{49152} - \frac{6656}{49152} = \frac{448}{49152}$ *summa excessus & defectus, cujus numerator est* $= d + a = 192 + 256$. *Ergo excessus est* $\frac{1}{256}$ *& defectus* $\frac{1}{192}$*: consequenter segmentum verum* $\frac{36}{256} = \frac{9}{64}$ *vel* $\frac{27}{192} = \frac{9}{64}$ *& ad quadratum diametri, ut* $\frac{9}{64} : 1 = 9 : 64$. *Est igitur diameter ad periph. ut* $8 : 25$ (§ 8.).

## PROBLEMA V.

§ 16. *Determinare tam excessum, quam defectum summæ, cujus numerator est præcise aggregatum ex multiplis denominatorum* $d + a$; *vel ex multiplo denominatoris unius & simplo alterius.*

Sit coefficiens denominatoris $d$, h. e. numerus indicans, quoties ille fuit sibimet additus, $= m$, & coefficiens denominatoris $a = l$: erit summa excessus & defectus $= \frac{md + la}{ad} = \frac{md}{ad} + \frac{la}{ad} = \frac{m}{a} + \frac{l}{d}$. Sit deinde numerator summæ $= md + a$; vel $d + la$, h.e. sit aggregatum ex multiplo denominatoris $d$ ac simplo denominatoris $a$, & vice versa: erit summa ipsa in 1mo casu $= \frac{md + a}{ad} = \frac{md}{ad} + \frac{a}{ad} = \frac{m}{a} + \frac{1}{d}$, & in secundo $= \frac{d + la}{ad} = \frac{d}{ad} + \frac{la}{ad} = \frac{1}{a} + \frac{l}{d}$; ex quo fluit

Regula: *Si numerator summæ est aggregatum ex multiplis denominatorum* d+a *ita, ut non possit resolvi in plura, quam in 2 multipla; vel si ille est aggregatum ex multiplo denominatoris* d *ac simplo denominatoris* a, *& vice versa; tunc coefficienti denominatoris* d, *vel si nullus adest, unitati subscribatur denominator quantitatis excessivæ & habebitur excessus: contra coefficienti denominatoris* a, *vel si nullus adest, unitati subscribatur denominator quantitatis defectivæ, & habebitur defectus.*

§. 17 Scholion I. *Ad illustrandum & confirmandum hoc problema, investigetur denuo lunula respondens diam. quadrato* $= 1$ *: Sit itaque ratio hujus quadrati ad lunulam excessiva, ut* $4 : 1\frac{2}{9} = 36 : 11$, *& defectiva ut* $4 : \frac{5}{8} = 32 : 5$*: erunt lunulæ per utramque rationem inventæ* $\frac{11}{36}$ *&* $\frac{5}{32} = \frac{352}{1152}$ *&* $\frac{180}{1152}$, *consequenter* $\frac{352}{1152} - \frac{180}{1152} = \frac{172}{1152}$ *summa excessus & defectus, cujus numerator est* $= 2d + 3a = 64 + 108$. *Iam cum coefficiens denominatoris* d *sit* 2, *& denominatoris* a *sit* 3; *scribatur sub* 2 *denominator quantitatis excessivæ, & habebitur excessus* $\frac{2}{36}$, *sub* 3 *autem scribatur denominator quantitatis defectivæ &*

*habe-*

habebitur defectus $\frac{3}{32}$. Sit deinde ratio excessiva, ut $4 : 1\frac{1}{4} = 16 : 5$ & defectiva, ut $4 : \frac{1}{5} = 20 : 1$ : erunt lunulæ $\frac{5}{16}$ & $\frac{1}{20} = \frac{100}{320}$ & $\frac{16}{320}$, consequenter $\frac{100}{320} - \frac{16}{320} = \frac{84}{320}$ summa excessus & defectus, cujus numerator est $= d † 4a = 20 † 64$. Iam cum denominator d nullo coefficiente sit affectus; subscribatur unitati denominator quantitatis excessivæ, ut prodeat excessus $\frac{1}{16}$; coefficienti 4 denominatoris a autem subscribatur denominator quantitatis defectivæ, ut habeatur defectus $\frac{4}{20}$. Sit denique ratio excessiva, ut $4 : 1\frac{3}{4} = 16 : 7$, & defectiva ut $4 : \frac{6}{7} = 28 : 6$ : erunt lunulæ $\frac{7}{16}$ & $\frac{6}{28} = \frac{196}{448}$ & $\frac{96}{448}$, consequenter $\frac{196}{445} - \frac{96}{448} = \frac{100}{448}$ summa excessus & defectus, cujus numerator est $= 3d † a = 84 † 16$. Iam cum coefficiens denominatoris d sit 3; subscribatur ei denominator quantitatis excessivæ, ut habeatur excessus $\frac{3}{16}$. Quoniam vero denominator a caret coefficiente, subscribatur unitati denominator quantitatis defectivæ, & habebitur defectus $\frac{1}{28}$. Cum itaque ablatis excessibus ex lunulis excessivis, vel additis defectibus ad defectivas, hic quoque prodeat lunula vera $= \frac{1}{4}$; palam est, excessus & defectus per problema præcedens exacte posse determinari.

§. 18. Scholion II. Sit diametri quadratum $= 1$, ratio ejus ad segmentum excessiva, ut $64 : 9\frac{2}{5} = 320 : 47$ & defectiva ut $64 : 8\frac{1}{4} = 256 : 33$ : erunt segmenta $\frac{47}{320}$ & $\frac{33}{256} = \frac{12032}{81920}$ & $\frac{10560}{81920}$ : consequenter $\frac{12032}{81920} - \frac{10560}{81920} = \frac{1472}{81920}$ summa excessus & defectus, cujus numerator est $= 2d † 3a = 512 † 960$. Ergo (per §. 16.) excessus est $\frac{2}{320}$ & defectus $\frac{3}{256}$ : consequenter segmentum verum $\frac{47}{320} - \frac{2}{320} = \frac{45}{320} = \frac{9}{64}$ vel $\frac{33}{256} † \frac{3}{256} = \frac{36}{256} = \frac{9}{64}$. Sit deinde ratio excessiva, ut $64 : 9\frac{1}{2} = 128 : 19$, & defectiva, ut $64 : 8\frac{1}{3} = 192 : 25$ : erunt segmenta $\frac{19}{128}$ & $\frac{25}{192} = \frac{3648}{24576}$ & $\frac{3200}{24576}$ : consequenter $\frac{3548}{24576} - \frac{3200}{24573} = \frac{448}{24576}$ summa excessus & defectus, cujus numerator est $= d † 2a = 192 † 256$. Ergo excessus est $\frac{1}{128}$ & defectus $\frac{2}{192}$ : consequenter segmentum verum $\frac{19}{128} - \frac{1}{128} = \frac{18}{128} = \frac{9}{64}$; vel $\frac{25}{192} † \frac{2}{192} = \frac{27}{192} = \frac{9}{64}$. Sit denique ratio excessiva $64 : 9\frac{2}{3} = 192 : 29$ & defectiva $64 : 8\frac{1}{2} = 128 : 17$ : erunt segmenta $\frac{29}{192}$ & $\frac{17}{128} = \frac{3712}{24576}$ & $\frac{3264}{24576}$, consequenter $\frac{3712}{24576} - \frac{3264}{24576} = \frac{448}{24576}$ summa excessus & defectus, cujus numerator est $= 2d † a = 256 † 192$. Ergo excessus est $\frac{2}{192}$ & defectus $\frac{1}{128}$ : consequenter segmentum verum $\frac{29}{192} - \frac{2}{192} = \frac{27}{192} = \frac{9}{64}$; vel $\frac{17}{128} † \frac{1}{128} = \frac{18}{128} = \frac{9}{64}$. Ergo segmentum est ad quadratum diametri ut $\frac{9}{64} : 1. = 9 : 64$, consequenter diameter ad peripheriam, ut $8 : 25$ (§. 8.).

§. 19. Scholion III. Sit diameter $= 1$, ratio ejus ad peripheriam excessiva, ut $8 : 25\frac{2}{3} = 24 : 77$, & defectiva, ut $8 : 24\frac{1}{5} = 40 : 121$ : erunt peripheriæ per utramque rationem inventæ $\frac{77}{24}$ & $\frac{121}{40} = \frac{3080}{960}$ & $\frac{2904}{900}$ : consequenter $\frac{3080}{960} - \frac{2904}{960} = \frac{176}{960}$ summa excessus & defectus, cujus numerator est $= 2d † 4a = 80 † 96$. Ergo (per §. 16.) excessus est $\frac{2}{24}$ & defectus $\frac{4}{40}$ : consequenter peripheria vera $\frac{77}{24} - \frac{2}{24} = \frac{75}{24} = \frac{25}{8}$; vel $\frac{121}{40} † \frac{4}{40} = \frac{125}{40} = \frac{25}{8}$. Sit deinde ratio excessiva,

*siva*, *ut* $8:26$ *& defectiva*, *ut* $8:24\frac{1}{9} = 72:217$: *erunt peripheriæ* $\frac{26}{8}$ *&* $\frac{217}{72} = \frac{1872}{576}$ *&* $\frac{1736}{576}$, *consequenter* $\frac{1872}{576} - \frac{1735}{576} = \frac{136}{576}$ *summa excessus & defectus*, *cujus numerator est* $= d \dagger 8a = 72 \dagger 64$. *Ergo excessus est* $\frac{1}{8}$ *& defectus* $\frac{8}{72}$, *consequenter peripheria vera* $\frac{26}{8} - \frac{1}{8} = \frac{25}{8}$; *vel* $\frac{217}{72} \dagger \frac{8}{72} = \frac{225}{72} = \frac{25}{8}$. *Sit denique ratio excessiva, ut* $8:25\frac{5}{7} = 56:181$, *& defectiva*, *ut* $8:24$; *erunt peripheriæ* $\frac{181}{56}$ *&* $\frac{24}{8} = \frac{1448}{448}$ *&* $\frac{1344}{448}$, *consequenter* $\frac{1448}{448} - \frac{1344}{448} = \frac{104}{448}$ *summa excessus & defectus*, *cujus numerator est* $= 6d \dagger a = 48 \dagger 56$. *Ergo excessus est* $\frac{6}{56}$ *& defectus* $\frac{1}{8}$: *consequenter peripheria vera* $\frac{175}{56} = \frac{25}{8}$; *vel* $\frac{24}{8} \dagger \frac{1}{8} = \frac{25}{8}$. *Est itaque diameter ad peripheriam*, *ut* $1 : \frac{25}{8} = 8:25$.

§. 20. *Corollarium.* Cum igitur per infinitas rationes numerator summæ excessus & defectus evadat aggregatum ex denominatoribus quantitatum excessivæ & defectivæ imo simplis, deinde multiplis, ac tandem ex multiplo denominatoris unius atque ex simplo alterius, & in quovis casu, ablato excessu ex quantitate excessiva vel addito defectu ad defectivam, constanter prodeat eadem ratio, nempe: quadrati diametri ad lunulam ut $4:1$; & ad segmentum, ut $64:9$; & ratio diametri ad peripheriam, ut $8:25$; evidens est quamlibet ex ratiocinio legitimo & ex principiis inconcussis, non autem ex combinatione arbitraria numerorum, esse deductam.

## THEOREMA.

§. 21. *Peripheriæ* Ludolphina, Metiana & Archimedea *peccant in excessu*: *1ma* $\frac{3}{200}$, *2da* $\frac{15}{904}$ *& 3tia* $\frac{9}{568}$ *diametri.*

*Demonstratio.* Quoniam ratio per quemcunque numerum 3tium multiplicata manet semper eadem; multiplicetur ratio *Ludolphi à Ceulen* $100:314$ per $2$, quo facto oritur $= 200:628$. Assumtis deinde diametro $= 1$ & ratione ejus ad peripheriam defectiva $8:24$, prodeunt peripheriæ $\frac{628}{200}$ & $\frac{24}{8} = \frac{5024}{1600}$ & $\frac{4800}{1600}$, consequenter $\frac{5024}{1600} - \frac{4800}{1600} = \frac{224}{1600}$ summa excessus & defectus (§. 2.), cujus numerator est $= 3d \dagger a = 24 \dagger 200$. Ergo (per §. 16.) excessus est $\frac{3}{200}$ & defectus $\frac{1}{8}$. *Quod erat 1mum.*

Ratio *Adriani Metii* $113:355$ multiplicata per $8$, dat $= 904:2840$, quæ cum defectiva $8:24$ efficit peripherias $\frac{2840}{904}$ & $\frac{24}{8} = \frac{22720}{7232}$ & $\frac{21696}{7232}$, consequenter $\frac{22720}{7232} - \frac{21696}{7232} = \frac{1024}{7232}$ summa excessus & defectus, cujus numerator est $= 15d \dagger a = 120 \dagger 904$. Ergo excessus est $\frac{15}{904}$ & defectus $\frac{1}{8}$. *Quod erat secundum.*

Ratio *Archimedis* $1 : 3\frac{10}{71}$ multiplicata per $568$ prodit $= 568:1784$, quæ cum defectiva $8:24$ manifestat peripheras $\frac{1784}{568}$ & $\frac{24}{8} = \frac{14272}{4544}$ & $\frac{13632}{4544}$, consequenter $\frac{14272}{4544} - \frac{13632}{4544} = \frac{640}{4544}$ summa excessus & defectus, cujus numerator est $= 9d \dagger a = 72 \dagger 568$. Ergo excessus est $\frac{9}{568}$ & defectus $\frac{1}{8}$. *Quod erat 3tium.*